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A possible reinterpretation of the Princeton superpipe data

By A. E. PERRY†, S. HAFEZ AND M. S. CHONG
Department of Mechanical and Manufacturing Engineering, University of Melbourne, Victoria 3010, Australia

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In experiments recently performed at Melbourne, Pitot-tube mean velocity profiles in a boundary layer disagreed with those obtained with hot wires. The standard MacMillan (1956) correction for the probe displacement effect and a correction for turbulence intensity were both required for obtaining agreement between the two sets of mean velocity data. We were thus motivated to reanalyse the Princeton superpipe data using the same two corrections. The result is a plausible conclusion that the superpipe is rough at the higher Reynolds numbers and its data follow the Colebrook (1939) formula for commerical pipes rather well. It also appears that the logarithmic law of the wall is valid, with a Kármán constant close to that found recently by Österlund (1999) from boundary layer measurements with a hot wire. The smooth regime in the pipe gave almost the same additive constant for the log-law as Österlund's. A comparison between the superpipe data and the pipe data of Perry, Henbest & Chong (1997) suggests that the conventional velocity defect law may be valid down to lower Reynolds numbers than concluded by Zagarola & Smits (1998).

1. Introduction

In recent boundary layer experiments at Melbourne, we found that the mean velocity profile measurements made with a Pitot tube disagreed with those obtained with hot wires. The Pitot-tube results had a characteristic 'kick up' above the log-law line in and around the buffer zone. Such a kick up was absent in the hot-wire results. In order to correct the Pitot-tube data so that they agree better with hot-wire data, we needed to apply two corrections. The first was the MacMillan (1956) correction that was originally developed for pipe flow. It corrects for the displacement effect of the probe due to mean shear. MacMillan tested a series of probes of different diameters and extrapolated the readings to a zero-diameter probe. The second correction accounts for the effect of turbulence intensity. Pitot-tube readings are affected by quadratic nonlinearities since pressures are averaged before velocities are evaluated, whereas hot-wire voltages are converted to velocities before any averaging is done, thus avoiding nonlinear problems. These corrections are significant only at viscous distances from the wall of less than 100, but the MacMillan correction was the major one throughout. The results of these corrections are described in Jones, Marusic & Perry (2001).

The 'kick up' mentioned earlier is also present in the superpipe data of Zagarola & Smits (1997). These authors interpreted it as part of a power law. Furthermore, the

† Sadly, Professor Perry died on 3 January 2001 during the preparation of this paper.

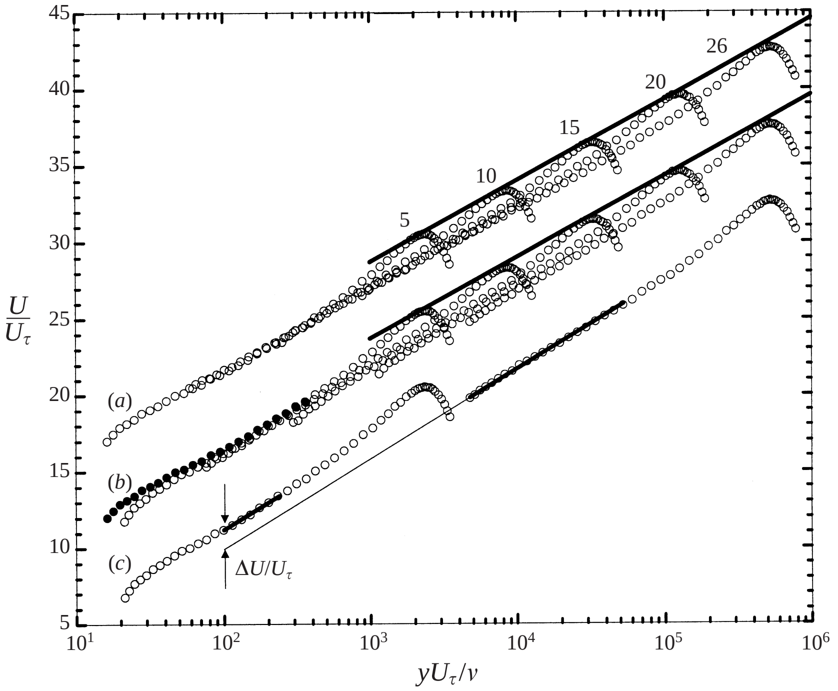

FIGURE 1. (*a*) Uncorrected superpipe data shifted up vertically by 5 units in U/U_τ. The smaller numbers in the profile sequence correspond with lower Reynolds number of the superpipe data, with 26 representing the highest Reynolds number. Profiles 5, 10, 15, 20 and 26 correspond to $a^+ = 2.344 \times 10^3$, 8.486×10^3, 3.288×10^4, 1.273×10^5 and 5.286×10^5 respectively. (*b*) Corrected superpipe data: • represents uncorrected data superimposed for comparsion. (*c*) Profile 5 is compared with profile 26 to show the range of $\Delta U/U_\tau$. Profiles are shifted down by 5 units. The slope of logarithmic lines in (*a*) and (*b*) is equal to 0.436, while in (*c*) it is 0.39.

superpipe data yield a quite different value of the Kármán constant than some recent results of Österlund (1999) for a boundary layer. Resolving these possible anomalies is the motivation for this work.

2. Analysis of data

A selection of uncorrected profiles measured in the superpipe is shown in figure 1(*a*). It is found that, for $y^+ > 100$, all profiles collapse approximately for a law of the form

$$\frac{U}{U_\tau} = \frac{1}{\kappa}\ln[y^+] + A + \frac{\Pi}{\kappa}h[y/a] \tag{2.1}$$

where U is the mean velocity, $y^+ = yU_\tau/\nu$ with y the distance from the wall, U_τ the friction velocity, and ν the kinematic viscosity, A is the smooth-wall additive constant and Π is a factor which some workers believe is dependent on the Reynolds number at low values; h is analogous to the Coles wake function in boundary layers and is a universal function of y/a where a is the pipe radius. Figure 1(*b*) shows the same data after application of the MacMillan correction and correction for turbulence intensity, the details of which are given in § 3 below. We decided to determine by a least-squares fit to the data the log-law constants by assuming that the law holds only for $y^+ > 100$ and $y/a < 0.1$. This law is shown as continuous lines in figure 1(*c*). One immediately sees that (2.1) needs to be modified to

$$\frac{U}{U_\tau} = \frac{1}{\kappa}\ln[y^+] + A - \frac{\Delta U}{U_\tau}[k_s^+] + \frac{\Pi}{\kappa}h[y/a]. \tag{2.2}$$

The displacement downwards of $\Delta U/U_\tau$ on the semi-log plot could be a roughness effect. In that case, $\Delta U/U_\tau$ is the Hama (1954) roughness function, which is a function of $k_s^+(\equiv k_s U_\tau/\nu)$, where k_s is the equivalent sand grain roughness. For a low Kármán number $a^+(\equiv aU_\tau/\nu)$, and hence low k_s^+, roughness effects are expected to be small. Tennekes & Lumley (1972) pointed out that for pipe flow the velocity defect law, based upon centreline and friction velocity, is independent of roughness for $k/a \ll 1$. This property applies to the present data after sufficient development length and is employed to deduce the value of κ. Furthermore, the value of A is obtained from the low Reynolds number data which gave an almost constant value. From the data one concludes that $\kappa = 0.39$ and $A = 4.42$ which compares well with $\kappa = 0.38$ and $A = 4.1$ as found by Österlund (1999) who carried out extensive zero-pressure-gradient boundary layer measurements using hot wires. It should be mentioned that log-laws with these two sets of numbers are indistinguishable for the range of y^+ data considered. The Reynolds number based on momentum thickness, R_θ was as high as 2.7×10^4 and the Kármán number $\delta^+ = 8000$ for Österlund's data, see also Österlund *et al.* (2000). The skin friction was determined using a fit to the buffer zone with a law derived from DNS data. The skin friction was also checked by an oil-film interferometry method. The values of κ and A differ slightly from ours because they fitted the log-law to data for $y^+ > 200$ and $y/\delta < 0.15$, where δ is the boundary layer thickness. The values of κ and A quoted by Zagarola & Smits (1998) are $\kappa = 0.436$ and $A = 6.15$.

In figure 1, the profile with the lowest Reynolds number has $a^+ = 2.344 \times 10^3$ and there is a short extent of the log-law. At lower Reynolds numbers there is little or no log-law region according to our strict criterion mentioned earlier; in practice, however, the data still appear to be logarithmic.

3. Details of corrections

The MacMillan correction for the mean shear is simply to add $0.15d_p$ to the y-coordinate where d_p is the Pitot-tube outer diameter. At $y/d_p < 2$ there is a wall proximity correction formulated by MacMillan but it was found not to be significant for the cases considered here. For the boundary layer data of Jones *et al.* (2001), we measured the turbulence intensity. For the superpipe such measurements do not exist and we assumed that the turbulence intensities followed the following similarity laws. For $y^+ > 50$

$$\frac{\overline{u_1^2}}{U_\tau^2} = 2.67 - 0.9\ln[y/a] - 6.06(y^+)^{-0.5} \tag{3.1}$$

proposed by Perry, Henbest & Chong (1986) for a pipe, and for $15 < y^+ \leqslant 50$

$$\frac{\overline{u_1^2}}{U_\tau^2} = (3.0 - 1.5\log[y^+/15])^2 \tag{3.2}$$

found by curve fitting some pipe data of Abell (1974). For $y^+ < 15$, one could use the data of Durst, Jovanović & Sender (1995) to formulate a correction. Here $\overline{u_1^2}$ is the mean square of the streamwise velocity fluctuation and the corrected mean velocity

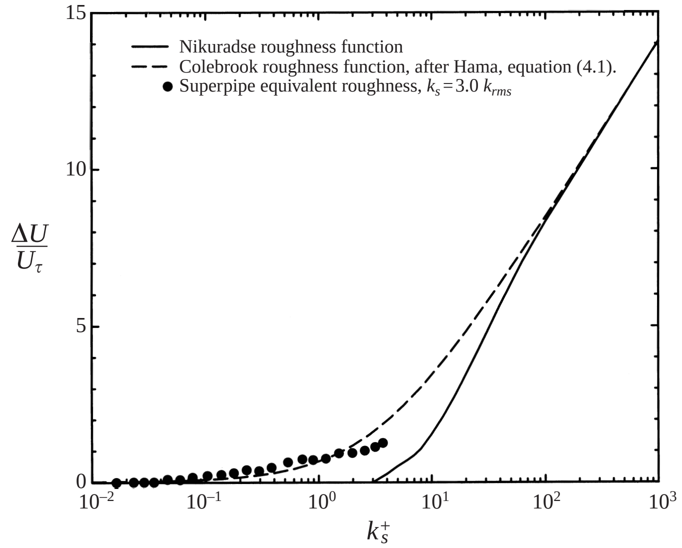

FIGURE 2. Roughness function.

U is found from

$$\frac{U}{U_\tau} = \left(\left(\frac{U_m}{U_\tau} \right)^2 - \left(\frac{\overline{u_1^2}}{U_\tau^2} \right) \right)^{0.5}. \tag{3.3}$$

U_m is the measured mean velocity. Here an overbar denotes a temporal mean. When these corrections were applied to the boundary layer results of Jones *et al.* (2001) the Pitot-tube results for $40 < y^+ < 80$ were shifted down by 0.45 units in U/U_τ but were still slightly above hot-wire results by about 0.2 units. Perhaps some further corrections for other components of turbulence are needed.

4. Effect of roughness

The effect of roughness on the superpipe data was pointed out by Barenblatt & Chorin (1998) who compared a profile from the superpipe with Nikuradse's (1932) smooth-pipe profile at about the same Reynolds number. Figure 2 shows the superpipe experimental values of the Hama (1954) roughness function as determined by us via equation (2.2) using the constants mentioned earlier, compared with the Hama equation

$$\frac{\Delta U}{U_\tau} = 5.66 \log[k_s^+ + 3.30] - 2.92 \tag{4.1}$$

derived from the Colebrook (1939) formula for 'natural roughness' or 'commercial roughness' such as the surfaces of wrought iron and cast iron. It can be seen that the comparison is quite reasonable. Figure 3 shows an expanded view. The value of k_s chosen was that quoted by Zagarola & Smits (1998). From the use of a comparator plate, they estimated the r.m.s. value of the surface roughness to be $k_{rms} = 0.15 \pm 0.03\,\mu\text{m}$ ($6.0 \pm 1.2\,\mu\text{in}$), see also Smits & Zagarola (1998). The equivalent sand grain roughness k_s was taken to be $3.0\,k_{rms}$ since it seems to fit the data in figures 2 and 3. Hama (1954) suggested $k_s = 5.0\,k_{rms}$, but this would hardly change figures 2 and 3 or their implications. The ratio used here between k_s and k_{rms} is also

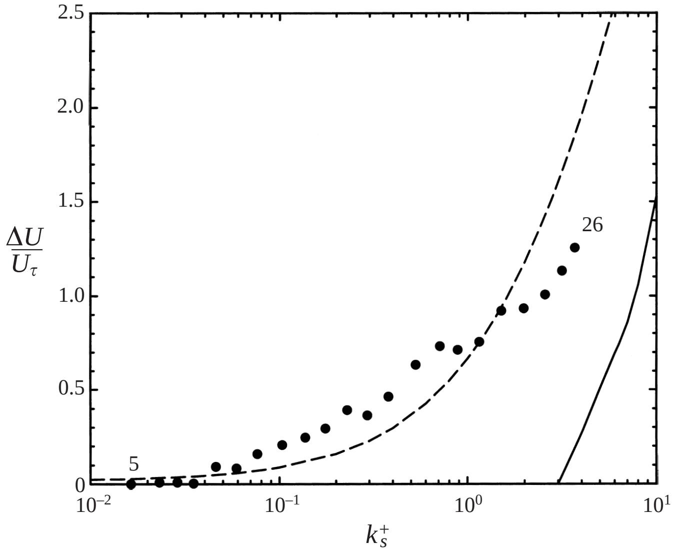

FIGURE 3. Roughness function, close-up. Points correspond to profiles 5 to 26.

of the same order as found by Acharya, Bornstein & Escudier (1986) who examined a variety of different artificial roughnesses in an attempt to simulate natural roughness. Although the shape of the Hama roughness functions varied from one geometry to another in the transitionally rough regime, the order of the effect was much the same as with the Colebrook formula. In figure 2 the mean of the Nikuradse (1933) sand grain roughness function data in the transitionally rough regime is also shown. As we were recently reminded by Bradshaw (2000), sand grain roughness behaviour has no relevance to natural roughness in the transitionally rough regime. However it is used for determining the equivalent sand grain roughness scale k_s of a given roughness by forcing the data to match with the sand grain formula in the fully rough regime, i.e. (4.1) with k_s^+ sufficiently large. Of course the formulae used here are based on slightly different values of κ, but this has only a second-order effect on the conclusions.

In figure 1(c), as already mentioned, are shown superimposed on the corrected data log-laws with $\kappa = 0.39$. Also shown as an envelope to the outer parts of the corrected and uncorrected profiles is a log-line with a slope corresponding to the Zagarola & Smits (1998) value of $\kappa = 0.436$. Since the MacMillan correction and turbulence intensity correction are negligible for this part of the flow, one might be tempted to say that the value of κ could be determined from the slope of this envelope. It avoids having to deal with the log-law region with associated corrections. Such logic would be correct if the wall were smooth. However, with roughness present, it is obvious that a false κ will be inferred using this method.

5. The velocity defect law

One of the most important laws established in pipe flow is the velocity defect law, which supports the principle of Reynolds number similarity upon which most turbulence theories and data interpretations are based. This has applications in all manner of flow situations, e.g. jets, wakes, mixing layers, boundary layers, etc. This principle states that, at sufficiently high Reynolds numbers, mean relative motions and energy-containing motions are independent of viscosity (or surface roughness) except

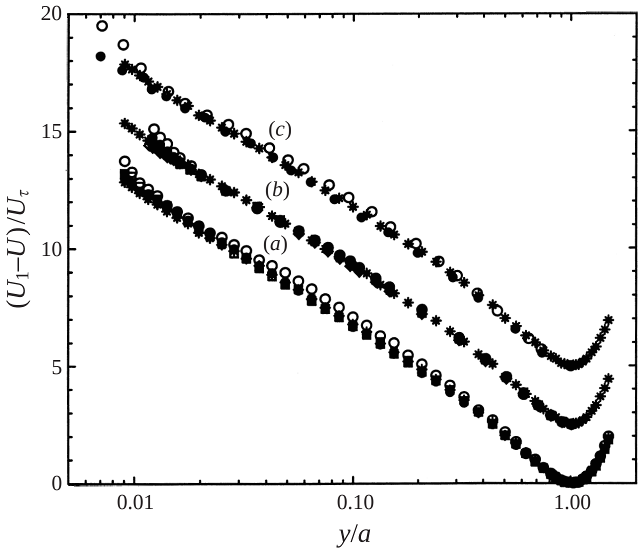

FIGURE 4. Defect plot. (*a*) Superpipe data, corrected profiles 5, 10, 15, 20 and 26. (*b*) Data of Henbest and (*c*) data of Abell, compared with profile 26 of the superpipe data. Profiles are shifted up by 2.5 units. Profile 26 is denoted by $*$ in (*a*), (*b*) and (*c*). Profile 5 denoted by $\circ$ only in (*a*).

in so far as viscosity (and surface roughness) may affect the boundary condition of the flow. Here, the Princeton data are compared with the smooth-pipe data of Perry, Henbest & Chong (1997), hereafter referred to as the Henbest data, and Abell (1974) to which MacMillan and turbulence corrections have been applied the same way as to the Princeton data. The Henbest data are already corrected by the MacMillan formula (private communications from Henbest) and so we apply only the correction for the turbulence intensity.

Figure 4(*a*) shows the superpipe profiles 5 to 26, given in figure 1 where a^+ ranges from 2.344×10^3 to 5.286×10^5, plotted in velocity defect form. All profiles collapse except profile 5. In fact it is found that all profiles collapse (including the ones omitted here for clarity) for $a^+ > 5000$. Figure 4(*b*) shows all the Henbest profiles for which $a^+ = 1600$ to 3800. They all collapse with the highest Reynolds number superpipe profile 26 where $a^+ = 5.286 \times 10^5$. Figure 4(*c*) shows the defect profiles of Abell. The lowest $a^+ = 2500$ profile collapses with profile 5 of the superpipe data but the highest $a^+ = 6200$ collapses with profile 26. Lack of collapse may indicate that the development length L or other entry conditions may be entering the problem. For the Henbest and Abell experiments, the boundary layer at the entrance immediately after the inlet contraction was tripped with a length of sandpaper glued to the pipe surface. The transition to turbulence in the superpipe was caused by background turbulence. The ratio L/d, where d is the pipe diameter, is 160 for the superpipe, 400 for the Henbest pipe, and 80 for the Abell pipe. It appears that low a^+ and the lower L/d give a slightly higher defect, whereas for $L/d = 400$, the defect law is valid unchanged for a^+ down to 1600. The reasons for this need to be explored, but in all cases for $a^+ > 5000$ the defect law works rather well and is independent of roughness. It may well be valid down to $a^+ = 1600$ which is the lowest of Henbest's data.

6. Conclusions

It appears that the classical description of turbulent pipe flow is valid provided we use the standard classical correction of MacMillan for the Pitot-tube readings and also a correction for turbulence intensity. The standard logarithmic law of the wall with $\kappa = 0.39$ and $A = 4.42$ appears to be valid for both boundary layers and pipes and the Colebrook formula for natural roughness is of the right order to explain the superpipe data. The classical Reynolds number invariance of the velocity defect law and its independence of the natural surface roughness is confirmed for $a^+ > 5000$. What is urgently needed next is an extensive set of mean velocity profiles in a pipe measured with a hot wire.

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